A sound transducer with a flat, flexible diaphragm working with bending waves

Daniela L. Manger

Manger Products, Industriestrasse 17, D-97638 Mellrichstadt, Germany

Summary: From the idea to a finally excellent working sound transducer a period of over twenty years was necessary. Nowadays it is possible to present a wide-band sound transducer working from 100 Hz to 35 kHz. It follows time-precise without any mechanical energy storage the incoming signal. The special structure of the flat pliable diaphragm works concentrically only with bending waves. Theoretical equations and measurements will be presented in comparison to the omnipresent piston loudspeaker. The advantages in perception and hearing will be mentioned for further research.

INTRODUCTION

In this paper the author will outline the basic theory of the bending-wave sound transducer as a resistive controlled driver in comparison to the piston loudspeaker. The fundamental descriptions has been made by Rice and Kellogg, comparing the elastic, resistance and mass controlled units for harmonic motion [1]:

- elastic control
  \[
  F = K \cdot x(t) \Rightarrow X = \frac{F}{K} = \text{const.}
  \]

- resistive control
  \[
  F = R \cdot u(t) \wedge u(t) = \frac{dx}{dt} \Rightarrow X = \frac{F}{\omega R} = \frac{F}{2\pi fR} \Rightarrow X \sim \frac{1}{f}
  \]

- mass control
  \[
  F = m \cdot a(t) \wedge a(t) = \frac{d^2x}{dt^2} \Rightarrow X = \frac{F}{\omega^2 m} = \frac{F}{4\pi^2 f^2 m} \Rightarrow X \sim \frac{1}{f^2}
  \]

The kinds of energy are for the elastic-controlled motion potential energy, for the resistive-controlled translation energy and for the mass-controlled kinetic energy. Further research on mechanics and structure-borne sound about a resistive behaviour led to the realisation of the resistive-controlled driver [2].

THEORETICAL DESCRIPTIONS

1. The mass-controlled driver

The theoretical description is known from the literature. The sound pressure is proportional to the acceleration of the piston, shown in the formula [3]:

\[
p(r, z, t) = \left( \frac{\rho_0 a^2}{4z} \right) A(t - \frac{r}{c}) \Rightarrow p(r, z, t) \sim A(t - \frac{r}{c})
\]

- \( p(r, z, t) = \) sound pressure at position \( r, z \) at time \( t \)
- \( c = \) speed of sound in air
- \( \rho_0 = \) density of air
- \( A(t-r/c) = \) acceleration at time \( (t-r/c) \)
- \( a = \) radius of circular piston
2. The resistive-controlled driver

A bending-wave transducer is a “resistive-controlled” device by implying that a force will produce a proportional velocity, which leads to an exact following sound pressure radiation of the incoming signal. To reach this the mechanical impedance has to be resistive and the electrical impedance of the voice coil is negligible. Further mechanical requirements in this case are: a large pliable circular plate with a matched impedance, so that no reflections will occur at the centre and outer boundaries. The motion of the plate can be described by the bending wave equation; i.e. membrane tensions (which would lead to non-linearities) must be absent. The radiation load of the air in front of the diaphragm can be neglected because of the higher weight of the diaphragm.

The mechanical impedance $Z_m$ of an infinite thin plate in vacuum is purely resistive, which is also true for higher frequencies in air [4]:

$$ Z_m = 8\sqrt{\frac{B'}{\rho h}} $$

$Z_m$ = mechanical impedance  
$B'$ = bending stiffness per unit width  
$h$ = thickness  
$\rho$ = density

The theoretical evaluations on the presented bending wave transducer were completed by Prof. Dr. Manfred Heckl in 1978 under taking into account the above mentioned requirements. The aim was to determine the sound radiation of such a configuration, with the time history of the sound pressure being of particular interest.

Figure 1: a circular plate excited along a ring with the used coordinates and variables for the calculation

The sound pressure radiated by a very large plane plate is given for the radial symmetric case by the formula [5]:

$$ p(r, z, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{j\omega t} \int_{0}^{\infty} \tilde{p}(k_r, \omega) J_0(k_r r) e^{-j\sqrt{k_0^2-k_r^2} z} \, dk_r \, d\omega. \quad (6) $$

If the plate is excited by a radial pressure distribution of the form $p_A(r, t)$, which is in the present case a ring-shaped excitation, we have

$$ \tilde{p}(k_r, \omega) = \frac{j \omega J_0(k_r a) \tilde{F}_0(\omega) \omega \rho_0}{4\pi^2 B(k_r^4 - k_0^4) \sqrt{k_0^2 - k_r^2}} \quad (7) $$

Because of the assumption that membrane tension is absent, we always have $k_r << k_0$ in the far field, i.e. for $k_0 > k_r$; therefore
The integral over $k_r$ can be obtained from a comparison with the calculation of the radiation from a small piston membrane, where we can derive the result of the bending wave transducer:

$$p(r, z, t) = \frac{j \rho_0}{4\pi^2 m''} \int_{-\infty}^{\infty} \frac{F_0(\omega)}{\sqrt{k_0^2 - k_r^2}} J_0(k_r r) e^{-j\sqrt{k_0^2 - k_r^2} z} e^{i\omega t} dk_r d\omega.$$  \hspace{1cm} (8)

This result says that the time history of the sound pressure near the symmetry axis corresponds exactly to the time history of the force acting on the plate, which is related by $F_0 = Z_m \cdot v$ with the diaphragm velocity and hence to the time history of the current in the voice coil, whereby the mass of the voice coil is negligible. Therefore, time histories of currents with sudden changes, e.g. square wave, are reproduced correctly in the radiated sound pressure. The time-lag due to sound wave propagation in the air is expressed by the argument $t - \sqrt{r^2 + z^2}/c$.

$$p(r, z, t) = \text{sound pressure at position } r, z \text{ at time } t$$

$r$ = perpendicular distance from the symmetry axis which passes through the centre of the ring  
$z$ = distance from the plate  
$t$ = time  
$\omega$ = integration variable (angular frequency)  
$J_0(\ldots)$ = Bessel function of order zero  
$j = \sqrt{-1}$  
$k_r$ = integration variable (wave number)  
$k_0 = \omega/c$  
$c$ = propagation speed of sound waves in air  
$p(\ldots)$ = wave number spectrum of the pressure  
$B$ = bending stiffness of the plate  
$m''$ = mass per unit area of the plate  
$k_B = 4\omega^2 m''/B$ = wave number of free bending waves

REALIZATION

Figure 2: Sketch of the Manger sound transducer
The presented bending wave transducer, which is protected as Manger Sound Transducer, has an electrodynamic motor working with two voice coils on one layer. Mechanical in serial operation and electrical in parallel it is possible compensating for emf which is generated by the voice coil movement. The inductivity of the voice coil is $18\mu$H, about 1000 times smaller than a conventional voice coil inductivity. The total weight of the voice coil is only 0.4 g. By this specifications the influence of the motor is only resistive in the interesting frequency range. The diaphragm diameter is 190 mm.

**MEASUREMENTS**

The dispersion of the bending waves on the diaphragm with the relation $\lambda_B \sim 1/\sqrt{f}$ can be shown with laser Doppler vibrometry. Measurements were made at various frequencies.

Figure 3: Laser Doppler vibrometry measurements at frequencies of 100, 200, 315, 1000, 3150, 10000 Hz

The time behaviour of the Manger sound transducer is shown in step response measurements. The input signal is an electrical step. The theoretical step response in air is shown in comparison to the step response of mass-controlled driver constructions.

Figure 4: Electrical input, theoretical ideal step response in air, step response of the Manger sound transducer, step response of a 3-way-piston loudspeaker measured in 1m

**CONCLUSION**

The proof for the time precise transduction using a flat, pliable circular plate as a resistive-controlled device was made by theoretical and practical means. The improvement of the aperiodic behaviour which is closed to the ideal transient response in air leads to a new awareness of sound reproduction in time and frequency. It enables a general improvement of sound reproduction and offers new ways of research on human perception and hearing.

**REFERENCES**